Collision-induced stimulated photon echo at the transition 0-1

V. A. Reshetov, E. N. Popov

Department of General and Theoretical Physics, Tolyatti State University, 14 Belorousskaya Street, 446667 Tolyatti, Russia

Abstract

The stimulated photon echo formed by the sequence of three laser excitation pulses in gaseous medium on the transition with the angular momentum change $J_a = 0 \rightarrow J_b = 1$ is considered with an account of elastic depolarizing collisions. The polarization properties of this echo may be potentially interesting for the purposes of data processing. It is shown, that with the polarization of the first pulse orthogonal to that of the second and third pulses the appearance of the echo signal will be entirely determined by the action of collisions. The existence of this echo is determined by the difference in the alignment $\Gamma_b^{(2)}$ and the orientation $\Gamma_b^{(1)}$ relaxation rates of the excited level.

1 Introduction

Since its first observation [1] photon echo was widely employed for the study of fast relaxation processes in various media (see, for example, some recent papers [2, 3, 4, 5]). It is also considered as a strong contender for implementation of quantum memory (see some recent reviews [6, 7] and the papers [8, 9, 10, 11, 12] in the special issue of Journal of Physics B, dedicated to quantum memories). Photon echo proved to be a versatile tool due to the large number of its modifications suitable for various purposes. The stimulated photon echo is formed on an optically-allowed transition $a \rightarrow b$ by the sequence of three resonant laser pulses. With the lower resonant level being some metastable level, the time interval between the excitation pulses may be rather long making such long-lived stimulated photon echoes attractive for purposes of data storage [13, 14, 15]. The modified stimulated photon echo [16, 17] is formed when the first two excitation pulses are in resonance with some optically-allowed transition $a \rightarrow b$, while the third pulse is in resonance with some adjacent transition $b \rightarrow c$. This echo signal is perfectly suited for the measurement of relaxation characteristics of the intermediate level b [18]. Another type of echo, which may be generated in a three-level system by three excitation pulses is the tri-level echo [19]. This echo is formed, when the first and the third excitation pulses are in resonance with one of the two optically-allowed transitions $a \rightarrow b$ sharing common level a, while the second pulse is in resonance with the other transition $a \rightarrow c$. Such echoes provide information about the relaxation characteristics of the optically-forbidden transition $b \rightarrow c$ [20]. A variety of novel photon echo schemes were elaborated for the implementation of quantum memories [21, 22, 23, 24, 25, 26, 27, 28, 29].

The collisional relaxation of photon echoes in gases was intensively studied both theoretically and experimentally starting from their first observation in [30] (see, for example, [31, 32, 33, 34]). Generally, the relaxation processes act to decrease the amplitude of the echo signals, however in some cases, when the conventional echo is locked, the relaxation may act to break the lock and to produce the relaxation-induced echo signals. Such collision-induced two-pulse photon echo was recently observed in the ytterbium vapour on the transition $J_a = 0 \rightarrow J_b = 1$ [35]. This echo appears due to the difference in the relaxation rates of the two components of the atomic dipole moment – collinear with the atomic velocity and perpendicular to it. The possibility of observation of the collision-induced stimulated photon echoes was indicated in papers [36, 37]. This type of echoes appear due to the difference in the relaxation rates of various multipole momenta of resonant levels. In the experiment [17] the intensity of the modified stimulated photon echo formed in ytterbium vapour on the transitions $J_a = 0 \rightarrow J_b = 1 \rightarrow J_c = 1$ was reduced greatly when the polarizations of all the three excitation pulses became collinear, which was interpreted in [38] as a collision-induced echo. Inspired by the experiment [35] we propose here a scheme, much simpler than in [36, 37], for observation of the collision-induced stimulated photon echo on a single transition $J_a = 0 \rightarrow J_b = 1$. In this case only three non-degenerate states are involved in the echo formation - the lower level and the two substates of the upper level, so that the stimulated photon echo generally contains all the types of echoes characteristic for three-level systems – the conventional two-level stimulated echo, the modified stimulated echo and the tri-level echo. Due to the simplicity of the level structure the choice of polarizations of the three excitation pulses provides enough freedom to separate any of these three echo types or to lock all of them enabling the appearance of the collision-induced echoes. Such polarization properties make this echo scheme potentially interesting for the purposes of data processing, for implementation of single q-bit quantum gates, in particular. However this echo scheme is not valid for data storage, since the ground state is non-degenerate ($J_a = 0$) and the storage time is limited by the rather short radiative lifetime of the excited level [39]. The appearance of the collision-induced echoes in this case is entirely determined by the difference in the alignment and orientation relaxation rates of the excited level, so its observation will provide information about depolarizing collisions.

In sections 2 and 3 the atomic dynamics under the action of short laser pulses and under the action of elastic depolarizing collisions and spontaneous radiational decay is described. In section 4 the general expression for the electric field strength of the echo signal is derived. In section 5 the polarization properties of the echo signal are analyzed in the absence of collisions and the possibilities of data processing is discussed, while in section 6 the polarization properties of the collision-induced stimulated photon echo are studied.

2 Atomic dynamics under the action of laser pulses

Let us consider the stimulated photon echo formation on the transition with the angular momentum change $J_a = 0 \to J_b = 1$ by the three resonant laser pulses with the durations T_1, T_2 and T_3, τ_1 being the time interval between the first and the second pulses and τ_2 – between the second and the third. The electric field strength of these pulses, propagating along the axis Z with the carrier frequency ω, may be put down as:

$$\mathbf{E}_n = e_n \mathbf{l}_n \exp\{-i(\omega t - kz)\} + c.c.,\ \ n = 1, 2, 3, \tag{1}$$

where e_n and $\mathbf{l}_n$ are the amplitudes and unit polarization vectors of the pulses. We shall consider the pulses to be much shorter, than all the relaxation times. Then the dynamics of the atom under the action of n-th laser pulse is governed by the equation for the slowly-varying atomic density matrix $\hat{\rho}$:

$$\dot{\hat{\rho}} = i\left[\hat{\Omega}_n, \hat{\rho}\right],\ \ \hat{\Omega}_n = \frac{|d| e_n}{\hbar}\left(\hat{g}_n + \hat{g}_n^{\dagger}\right). \tag{2}$$

Here $d = d(J_a J_b)$ is the reduced matrix element of the electric dipole moment operator for the transition $J_a = 0 \to J_b = 1$, while $\hat{g}_n = (\hat{\mathbf{g}}\mathbf{l}_n^*)$, where $\hat{\mathbf{g}}$ is the dimensionless electric dipole moment operator for this transition, the matrix elements of its circular components, expressed through Wigner 3J-symbols, are as follows:

$$(\hat{g}_n)^{ab}_{0q} = \frac{1}{\sqrt{3}}(l_{n,-q})^*, \tag{3}$$

where $l_{n,q}$ are the circular components of the unit polarization vector of the n-th laser pulse. The solution of the equation (2) may be expressed through the evolution operator $\hat{S}_n$:

$$\hat{\rho}(T_n) = \hat{S}_n \hat{\rho}(0) \hat{S}_n^{\dagger},\ \ \hat{S}_n = \exp\left(i\int_0^{T_n} \hat{\Omega}_n dt\right), \tag{4}$$

$\hat{\rho}(0)$ and $\hat{\rho}(T_n)$ being the atomic density matrices at some point z of the gaseous medium before and after the n-th laser pulse passes through this point. The explicit expression of the evolution operator may be obtained by means of expansion of the exponent function in Taylor series and by diagonalization of the operator $\hat{\Omega}_n$. Defining the area of the n-th excitation pulse by the relation

$$\theta_n = \frac{2|d|}{\sqrt{3}\hbar}\int_0^{T_n} e_n dt, \tag{5}$$

we immediately obtain the matrix elements of the evolution operator involving the lower atomic level a:

$$(\hat{S}_n)^{aa}_{00} = \cos\left(\frac{\theta_n}{2}\right), (\hat{S}_n)^{ab}_{0q} = i\sin\left(\frac{\theta_n}{2}\right)(l_{n,-q})^*. \tag{6}$$

In order to obtain the matrix elements $(\hat{S}_n)^{bb}$ of the evolution operator referring to the upper state b, we note that the operator $\hat{g}_n^{\dagger}\hat{g}_n$ has the following three eigenvectors: the "bright" state

$$|b_n >= \sum_{q=\pm 1} l_{n,-q}|J_b = 1, m_b = q > \tag{7}$$

with non-zero eigenvalue, the "dark" state

$$|\tilde{b}_n >= \sum_{q=\pm 1} s_{n,-q}|J_b = 1, m_b = q > \tag{8}$$

with zero eigenvalue, where $\mathbf{s}_n$ is the unit vector in the plane XY, which is orthogonal to the vector $\mathbf{l}_n$ ($\mathbf{s}_n\mathbf{l}_n^* = 0$), and the state $|J_b = 1, m_b = 0 >$, which is not involved into interactions at any polarization of laser pulse and may be neglected. Then, for the matrix elements of the part $(\hat{S}_n)^{bb}$ of the evolution operator we obtain:

$$(\hat{S}_n)^{bb}_{qq'} = s_{n,-q}s^*_{n,-q'} + l_{n,-q}l^*_{n,-q'}\cos\left(\frac{\theta_n}{2}\right). \tag{9}$$

3 Atomic dynamics under the action of depolarizing collisions

The evolution of the density matrix elements in the time interval between the pulses is determined by the frequency detuning $\Delta = kv_z - \omega + \omega_0$, including Doppler shift kv_z, and by the irreversible relaxation. Here we shall take into account the two most rapid relaxation processes - the spontaneous radiation decay of the excited level and the elastic depolarizing collisions, which do not change the atomic velocities but give rise to the transitions between various Zeeman sublevels of atomic resonant levels. Then, the equations for the irreducible density matrix components ($\alpha, \beta = a, b$)

$$\psi^{\alpha\beta}_{kq} = \sqrt{2J_\alpha + 1}\sum_{\mu,\nu}(-1)^{J_\alpha - \mu}\begin{pmatrix} J_\alpha & J_\beta & k \\ \mu & -\nu & q \end{pmatrix}\rho^{\alpha\beta}_{\mu\nu}$$

are as follows [34]:

$$\dot{\psi}^{bb}_{kq} = -\gamma_b^{(k)}\psi^{bb}_{kq}, \tag{10}$$

$$\dot{\psi}^{aa}_{kq} = -\gamma_a^{(k)}\psi^{aa}_{kq} + \Gamma^{(k)}_{ab}\psi^{bb}_{kq}, \tag{11}$$

$$\dot{\psi}^{ab}_{kq} = -[\gamma^{(k)} + i(\Delta^{(k)} - \Delta)]\psi^{ab}_{kq}, \tag{12}$$

where

$$\gamma^{(k)}_{a,b} = \gamma^{(0)}_{a,b} + \Gamma^{(k)}_{a,b},$$

$$\gamma^{(k)} = \frac{1}{2}\left(\gamma_a^{(0)} + \gamma_b^{(0)}\right) + \Gamma^{(k)},$$

$$\Gamma_{ab}^{(k)} = (-1)^{k+1+J_a+J_b}\gamma_{ab}\times$$
$$\sqrt{(2J_a+1)(2J_b+1)}\begin{Bmatrix} J_a & J_a & k \\ J_b & J_b & 1 \end{Bmatrix}.$$

Here $1/\gamma_a^{(0)}$ and $1/\gamma_b^{(0)}$ are the lifetimes of atomic levels a and b due to spontaneous radiation decay, $\Gamma_a^{(k)}$ and $\Gamma_b^{(k)}$ describe the relaxation of the levels a and b due to the action of the elastic depolarizing collisions, ($\Gamma_a^{(0)} = \Gamma_b^{(0)} = 0$, since the elastic collisions do not alter the population of the levels), $\Gamma^{(k)}$ and $\Delta^{(k)}$ describe the relaxation of the irreducible components of the optical coherence matrix due to elastic depolarizing collisions, and $1/\gamma_{ab}$ is the partial lifetime of the upper level b due to the spontaneous radiation transitions to the lower level a, while Wigner 6J-symbols are denoted in a usual way [40]. In case of transition $J_a = 0 \to J_b = 1$ in ytterbium vapour a is the ground state, so $\gamma_a^{(0)} = 0$, $\gamma_b^{(0)} = \gamma_{ab} = \gamma$. The solution of the equations (10)-(12) is easily obtained and the elements $\rho_{\mu\nu}^{\alpha\beta}(t)$ of the density matrix in the original basis of Zeeman states, expressed through these solution by means of inverse transformation of irreducible density matrix components are as follows:

$$\rho_{00}^{aa}(t) = e^{-\gamma t}\rho_{00}^{aa}(0), \tag{13}$$

$$\rho_{0q}^{ab}(t) = e^{-(\gamma^{(1)}-i\delta)t}\rho_{0q}^{ab}(0), \tag{14}$$

$$\rho_{qq'}^{bb}(t) = e^{-\gamma t}\sum_{\sigma,\sigma'} D_{qq'}^{\sigma\sigma'}(t)\rho_{\sigma\sigma'}^{bb}(0), \tag{15}$$

$$D_{qq'}^{\sigma\sigma'}(t) = \delta_{q\sigma}\delta_{q'\sigma'} + R_{qq'}^{\sigma\sigma'}(t). \tag{16}$$

where $\delta = \Delta - \Delta^{(1)}$,

$$R_{1,1}^{1,1}(t) = R_{-1,-1}^{-1,-1}(t) = \frac{1}{6}h_b^{(2)}(t) + \frac{1}{2}h_b^{(1)}(t),$$

$$R_{-1,-1}^{1,1}(t) = R_{1,1}^{-1,-1}(t) = \frac{1}{6}h_b^{(2)}(t) - \frac{1}{2}h_b^{(1)}(t),$$

$$R_{1,-1}^{1,-1}(t) = R_{-1,1}^{-1,1}(t) = h_b^{(2)}(t),$$

$$h_b^{(k)}(t) = 1 - e^{-\Gamma_b^{(k)}t}, \tag{17}$$

while all the other elements of relaxation matrix $R_{qq'}^{\sigma\sigma'}(t)$ are zero.

4 Stimulated photon echo formation

Initially the atom is at its ground state a. The first laser pulse creates atomic coherence on the transitions $m_a = 0 \to m_b = q = \pm 1$, which is described by the non-diagonal elements of the atomic density matrix according to (4):

$$\rho_{0q}^{ab}(T_1) = (\hat{S}_1)_{00}^{aa}(\hat{S}_1^{\dagger})_{0q}^{ab}. \tag{18}$$

The evolution of these non-diagonal density matrix elements, which are responsible for the echo formation, in the time interval between the pulses is determined by the equation (14), so that at the instant of time $t - z/c - T_1 = \tau_1$, when the second excitation pulse arrives at point z of the gaseous medium,

$$\rho^{ab}_{0q}(\tau_1) = \rho^{ab}_{0q}(T_1) e^{-\gamma^{(1)}\tau_1 + i\delta\tau_1}. \tag{19}$$

The second laser pulse creates the modulated velocity distributions, proportional to $e^{-i\delta\tau_1} \sim e^{-ikv_z\tau_1}$, of the atoms at the upper and lower levels according to the solution of the equation (4):

$$\rho^{aa}_{00}(T_2) = \sum_q (\hat{S}_2)^{ab}_{0q} \rho^{ba}_{q0}(\tau_1) (\hat{S}^\dagger_2)^{aa}_{00}, \tag{20}$$

$$\rho^{bb}_{qq'}(T_2) = \sum_{q''} (\hat{S}_2)^{bb}_{qq''} \rho^{ba}_{q''0}(\tau_1) (\hat{S}^\dagger_2)^{ab}_{0q'}. \tag{21}$$

After the passage of the second pulse through the medium the relaxation of the these modulated velocity distributions is described by the equations (13) and (15), so that at the instant of time $t - z/c - T_1 - \tau_1 - T_2 = \tau_2$, when the third excitation pulse arrives at point z of the medium:

$$\rho^{aa}_{00}(\tau_2) = e^{-\gamma\tau_2} \rho^{aa}_{00}(T_2), \tag{22}$$

$$\rho^{bb}_{qq'}(\tau_2) = e^{-\gamma\tau_2} \sum_{\sigma,\sigma'} D^{\sigma\sigma'}_{qq'}(\tau_2) \rho^{bb}_{\sigma\sigma'}(T_2). \tag{23}$$

The third laser pulse creates atomic coherence on the transitions $m_a = 0 \to m_b = q = \pm 1$ from these modulated velocity distributions (22) and (23) according to (4):

$$\rho^{ab}_{0q}(T_3) = [\rho^{ab}_{0q}(T_3)]_a + [\rho^{ab}_{0q}(T_3)]_b, \tag{24}$$

$$[\rho^{ab}_{0q}(T_3)]_a = (\hat{S}_3)^{aa}_{00} \rho^{aa}_{00}(\tau_2) (\hat{S}^\dagger_3)^{ab}_{0q},$$

$$[\rho^{ab}_{0q}(T_3)]_b = \sum_{q',q''} (\hat{S}_3)^{ab}_{0q'} \rho^{bb}_{q'q''}(\tau_2) (\hat{S}^\dagger_3)^{bb}_{q''q}.$$

The evolution of these density matrix elements (24) after the action of the third laser pulse is determined by the equation (14), so that at the instant of time $t' = t - z/c - T_1 - \tau_1 - T_2 - \tau_2 - T_3$:

$$\rho^{ab}_{0q}(t') = \rho^{ab}_{0q}(T_3) e^{-\gamma^{(1)}t' + i\delta t'}. \tag{25}$$

The electric field strength of the stimulated photon echo signal

$$\mathbf{E}^e = \mathbf{e}^e(t') \exp\{-i(\omega t - kz)\} + c.c.,$$

is obtained from the Maxwell equations in a usual way:

$$\mathbf{e}^e(t') = ie_0 \int f(\mathbf{v}) d\mathbf{v} Tr\{\hat{\rho}(t') \hat{\mathbf{g}}\}, \tag{26}$$

where $e_0 = 2\pi\omega L n_0 |d|/c$, L is the length of the gaseous medium, n_0 is the concentration of resonant atoms, $f(\mathbf{v})$ is the Maxwell velocity distribution function, while the atomic density matrix $\hat{\rho}(t')$ at the instant of time t' is determined by (25). Henceforth we shall neglect the action of irreversible relaxation during the echo pulse as well as it was neglected during the excitation pulses and confine ourselves to the case of exact resonance. After the successive substitution of (18)-(25) in (26) with an account of the expressions (6) and (9) for the evolution operator the projections $e_n^e(t') = \mathbf{e}^e(t')\mathbf{e}_n^*$ of the echo amplitude on the two orthonormal vectors $\mathbf{e}_n$ $(n = 1, 2)$ in the plane of polarization XY may be expressed by the following equation:

$$e_n^e(t') = e_0 \frac{1}{\sqrt{3}} e^{-2\gamma^{(1)}\tau_1 - \gamma\tau_2} I_e(t') G_n^e, \tag{27}$$

where factor

$$I_e(t') = e^{-\frac{1}{4}k^2 u^2 (t'-\tau_1)^2} \tag{28}$$

describes the temporal shape of the echo pulse, u being the atomic thermal velocity, while factor

$$G_n^e = \frac{1}{2}\sin(\theta_1)(F_n^a + F_n^b + F_n^c) \tag{29}$$

describes the polarization properties of the echo signal. This factor contains three terms, the first two of them F_n^a and F_n^b are responsible for the contributions of the coherence created by the first two exciting pulses at the atomic levels a and b respectively:

$$F_n^a = \frac{1}{4}\sin(\theta_2)\sin(\theta_3)(\mathbf{l}_1^*\mathbf{l}_2)(\mathbf{e}_n^*\mathbf{l}_3), \tag{30}$$

$$F_n^b = \sin\left(\frac{\theta_2}{2}\right)\sin\left(\frac{\theta_3}{2}\right) U_2(\mathbf{l}_3, \mathbf{l}_1) U_3^*(\mathbf{e}_n, \mathbf{l}_2), \tag{31}$$

$$U_k(\mathbf{a}, \mathbf{b}) = (\mathbf{a}\mathbf{s}_k^*)(\mathbf{s}_k\mathbf{b}^*) + \cos\left(\frac{\theta_k}{2}\right)(\mathbf{a}\mathbf{l}_k^*)(\mathbf{l}_k\mathbf{b}^*), \tag{32}$$

while the third term F_n^c is entirely determined by the action of elastic depolarizing collisions.

5 Photon echo polarization in the absence of collisions

In the absence of elastic depolarizing collisions $\Gamma_b^{(1)} = \Gamma_b^{(2)} = 0$, the relaxation matrix $R_{qq'}^{\sigma\sigma'}(\tau_2)$ in (16) becomes zero and the echo signal is determined only by the terms F_n^a and F_n^b in the expression (29). In the general expression (29) three different types of echo signals may be distinguished.

The first one occurs when all the excitation pulses are identically polarized: $\mathbf{l}_1 = \mathbf{l}_2 = \mathbf{l}_3 = \mathbf{e}_1$. All the three pulses couple the non-degenerate ground state

to a single substate $|b_1>$ ("bright state") of the upper level (7). In this case the echo signal represents itself a conventional two-level stimulated photon echo, its polarization coincides with that of the excitation pulses:

$$G_1^e = \frac{1}{4}\sin(\theta_1)\sin(\theta_2)\sin(\theta_3),\ G_2^e = 0, \tag{33}$$

and the maximum echo amplitude takes place at

$$\theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}.$$

The second type of echo occurs when the first two excitation pulses are identically polarized: $\mathbf{l}_1 = \mathbf{l}_2 = \mathbf{e}_1$, while the polarization of the third pulse is orthogonal to that of the first two: $\mathbf{l}_3 = \mathbf{e}_2$. The first two pulses create the non-thermal velocity distributions of the atoms at the ground state and at the "bright state" $|b_1>$ of the upper level, however the third pulse couples the ground state to the initially unpopulated orthogonal "dark" state $|\tilde{b}_1>$ of the upper level (8), so that only the atoms at the ground state contribute to the echo signal. In this case the echo signal represents itself a "modified" three-level stimulated photon echo, its polarization coincides with that of the third excitation pulse:

$$G_2^e = \frac{1}{8}\sin(\theta_1)\sin(\theta_2)\sin(\theta_3),\ G_1^e = 0, \tag{34}$$

while its amplitude constitutes just the half of that of the stimulated photon echo signal of the first type, in all the other ways the properties of these first two types of echo signals being identical.

The third type of echo occurs when the polarizations of the first two excitation pulses are orthogonal to each other, while the polarization of the third pulse coincides with that of the first one: $\mathbf{l}_1 = \mathbf{l}_3 = \mathbf{e}_1$, $\mathbf{l}_2 = \mathbf{e}_2$. In this case the first two pulses create the coherence on the transition between the "bright" $|b_1>$ and the "dark" $|\tilde{b}_1>$ states of the upper level. The third pulse, which couples the ground state to the "bright" state $|b_1>$, transfers this coherence to the transition between the ground state and the "dark" state $|\tilde{b}_1>$ producing the echo signal on this transition, so that the echo polarization coincides with that of the second pulse:

$$G_2^e = \frac{1}{2}\sin(\theta_1)\sin\left(\frac{\theta_2}{2}\right)\sin\left(\frac{\theta_3}{2}\right),\ G_1^e = 0. \tag{35}$$

This echo type substantially differs from the first two types, its maximum amplitude takes place at

$$\theta_1 = \frac{\pi}{2},\ \theta_2 = \theta_3 = \pi,$$

when the signals of the first two types disappear. Generally the stimulated photon echo contains all the three types of signals, so that its polarization depends on the areas of the excitation pulses.

Finally, when the polarizations of the first two excitation pulses are orthogonal to each other and the polarization of the third pulse coincides with that of the second one: $\mathbf{l}_1 = \mathbf{e}_1$, $\mathbf{l}_2 = \mathbf{l}_3 = \mathbf{e}_2$, the stimulated echo signal does not appear in the absence of collisions:

$$F_n^a = F_n^b = 0.$$

In this case the echo signal will be entirely determined by the action of the elastic depolarizing collisions, so it may be identified as collision-induced stimulated photon echo.

Let us now consider the excitation pulses with small areas $\theta_n \ll 1$ and arbitrary polarizations $\mathbf{l}_n$ $(n = 1, 2, 3)$. Then we obtain from (29)-(32) the following expression for the echo polarization (unnormalized):

$$G_n^e = \frac{1}{8}\theta_1\theta_2\theta_3 \left\{ (\mathbf{l}_1^*\mathbf{l}_2)(\mathbf{e}_n^*\mathbf{l}_3) + (\mathbf{l}_1^*\mathbf{l}_3)(\mathbf{e}_n^*\mathbf{l}_2) \right\}.$$

If the second and the third excitation pulses are weak classical pulses polarized orthogonally to each other: $\mathbf{l}_2 = \mathbf{e}_1$, $\mathbf{l}_3 = \mathbf{e}_2$, while the first excitation pulse is a single-photon pulse with arbitrary polarization

$$\mathbf{l}_1 = \xi_1\mathbf{e}_1 + \xi_2\mathbf{e}_2,\ |\xi_1|^2 + |\xi_2|^2 = 1,$$

then the echo pulse is a single-photon pulse with the polarization

$$\mathbf{l}_e = \xi_2^*\mathbf{e}_1 + \xi_1^*\mathbf{e}_2.$$

So, the considered echo scheme may implement the single q-bit quantum NOT-gate with complex conjugation, which transforms the q-bit (ξ_1, ξ_2), encoded in the polarization state of the first single-photon excitation pulse, to the q-bit (ξ_2^*, ξ_1^*), encoded in the polarization state of the single-photon echo pulse.

6 Polarization properties of the collision-induced stimulated photon echo

The collision-induced stimulated photon echo is formed by the three excitation pulses, when the polarizations of the first two of them are orthogonal to each other and the polarization of the third pulse coincides with that of the second one: $\mathbf{l}_1 = \mathbf{e}_1$, $\mathbf{l}_2 = \mathbf{l}_3 = \mathbf{e}_2$. Without loss of generality the circular components of the two unit orthonormal vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ in the plane XY of polarization of the excitation pulses may be determined by the only one real parameter α:

$$e_{1,q} = \cos(\alpha)\delta_{q,-1} - \sin(\alpha)\delta_{q,1}, \tag{36}$$

$$e_{2,q} = \sin(\alpha)\delta_{q,-1} + \cos(\alpha)\delta_{q,1}. \tag{37}$$

The value $\alpha = 0$ corresponds to the circular orthogonal polarizations – first pulse right-circularly polarized $e_{1,q} = \delta_{q,-1}$, second and third – left-circularly

polarized $e_{2,q} = \delta_{q,1}$, while the value $\alpha = \pi/4$ corresponds to the linear orthogonal polarizations – first pulse polarized along the axis X, second and third – along the axis Y. After the substitution of (36)-(37) in the corresponding expressions for the evolution operators the factor (29) in the echo amplitude becomes as follows:

$$G_n^e = \frac{1}{4}\sin(\theta_1)\sin\left(\frac{\theta_2}{2}\right)\sin\left(\frac{\theta_3}{2}\right)h(\tau_2)e_n^c, \tag{38}$$

$$h(\tau_2) = \left(e^{-\Gamma_b^{(2)}\tau_2} - e^{-\Gamma_b^{(1)}\tau_2}\right), \tag{39}$$

$$e_1^c = \sin^2(2\alpha),\ e_2^c = -\frac{1}{2}\sin(4\alpha)\cos\left(\frac{\theta_3}{2}\right). \tag{40}$$

In the case of linearly orthogonally polarized excitation pulses ($\alpha = \pi/4$) the echo signal is also linearly polarized like the first pulse: $e_1^c = 1$, $e_2^c = 0$, while in the case of circularly orthogonally polarized excitation pulses ($\alpha = 0$) the echo signal does not appear: $e_1^c = e_2^c = 0$. In the case of circularly polarized pulses the collisional echo does not appear due to the symmetry with respect to rotation around the quantization axis. Such a symmetry, as it was shown in [41], forbids the collisional conversion of coherence $\rho^{bb}_{-1,1}(T_2)$ into $\rho^{bb}_{1,-1}(\tau_2)$ in the equation (23) (or $\rho^{bb}_{1,-1}(T_2)$ into $\rho^{bb}_{-1,1}(\tau_2)$), which is needed for the formation of collisional echo in this case. However, in the case of elliptically polarized pulses this symmetry is broken and the collisional echo, which is induced by the collisional transfer of populations $\rho^{bb}_{-1,-1}(T_2)$ to $\rho^{bb}_{1,1}(\tau_2)$ and $\rho^{bb}_{1,1}(T_2)$ to $\rho^{bb}_{-1,-1}(\tau_2)$ in this case, becomes allowed.

The dependence of the collision-induced echo amplitude on the time interval τ_2 between the second and the third excitation pulses differs drastically from that of the conventional stimulated photon echo. The latter decreases exponentially with the increase of τ_2, while the amplitude of the collision-induced echo increases first from its zero value with the increase of τ_2, then obtains its maximum at

$$\tau_{2m} = \frac{\ln(1+\Gamma_b^{(2)}/\gamma) - \ln(1+\Gamma_b^{(1)}/\gamma)}{\Gamma_b^{(2)} - \Gamma_b^{(1)}}, \tag{41}$$

and then it exponentially decreases with the further increase of τ_2. The mere existence of the collision-induced stimulated photon echo is determined by the difference in the alignment $\Gamma_b^{(2)}$ and the orientation $\Gamma_b^{(1)}$ relaxation rates of the excited level. At small τ_2 intervals ($|\Gamma_b^{(2)} - \Gamma_b^{(1)}|\tau_2 \ll 1$) the echo amplitude is proportional to $|\Gamma_b^{(2)} - \Gamma_b^{(1)}|$.

The collision-induced stimulated photon echo, proposed in the present paper, has much in common with the two-pulse collision-induced photon echo, observed in ytterbium vapour [35], though there are some essential differences. First, the two-pulse echo is determined by the difference in the two relaxation characteristics of the resonant transition, while the stimulated echo is determined by the difference in the relaxation characteristics of the excited level. Second, for the

existence of the two-pulse collision-induced echo the dependence of the relaxation characteristics on the orientation of atomic velocity is mandatory, while for the existence of the stimulated collision-induced echo such dependence is insignificant. Third, at small intervals between the excitation pulses the amplitude of the two-pulse echo is proportional to the squared difference in two relaxation rates of the resonant transition, while the amplitude of the stimulated echo is just proportional to the difference in two relaxation rates of the excited level, which promises the greater effect for the stimulated echo, than for the two-pulse echo.

7 Conclusions

In the case of simple transition with the angular momentum change $J_a = 0 \to J_b = 1$ only three atomic states contribute to the stimulated photon echo formation – the non-degenerate ground state and the two substates of the excited level. With the proper choice of polarizations of excitation pulses three types of echo signals may be distinguished in the absence of collisions: the conventional two-level stimulated echo, the modified three-level stimulated echo and the tri-level echo. Generally the resulting signal contains all the three types of echoes, so that the echo polarization depends on the areas of excitation pulses. Such polarization properties may be employed for the implementation of quantum NOT-gate. If the polarizations of the second and the third excitation pulses are identical and orthogonal to that of the first one, the echo signal is entirely determined by the action of elastic depolarizing collisions, so it may be identified as collision-induced stimulated photon echo. The existence of this echo is determined by the difference in the alignment $\Gamma_b^{(2)}$ and the orientation $\Gamma_b^{(1)}$ relaxation rates of the excited level.

In conclusion it should be mentioned, that although the photon echo formation was treated here under the assumption of narrow spectral line, when the durations of all excitation pulses were considered to be much shorter than the inhomogeneous relaxation time, all the results remain true also in the opposite case of broad spectral lines, only the dependence on the pulse areas becomes more complicated than just sine or cosine functions.

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